Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Spin Variations in Slender Entry Vehicles during Rolling Trim

WILLIAM J. BOOTLE*
Avco Systems Division, Wilmington, Mass.

Nomenclature

c.g.	= center of gravity
c.p.	= center of pressure
C_{m_q}	= damping in pitch derivative
$C_{m\alpha}$	= pitching moment coefficient derivative
$C_{N\alpha}$	= normal force coefficient derivative
D	= vehicle reference diameter
$I_{x,y,z}$	= moment of inertia in roll pitch, yaw
I_{xy}, I_{xz}	= products of inertia
K_{cr}	= critical double asymmetry product for steady resonance
p	= vehicle roll rate
$\dot{p}$	= roll acceleration
q	= dynamic pressure
R	= amplification ratio
S	= vehicle reference area
t	= time
V_E	= vehicle entry velocity
$\bar{X}$	= dimensional value of static margin
α_T, β_T	= static trim angles in pitch, yaw
β	= density lapse rate for an exponential atmosphere
γ_E	= flight path angle at entry
Δr	= radial c.g. offset
$\Delta y, \Delta z$	= vehicle c.g. offset along y, z axes
ϵ	= principal axis misalignment angle
λ	= a nondimensional parameter defined in the text
ρ	= atmospheric density
ω_c	= vehicle critical frequency $(C_{m\alpha}qSD/I_y - I_x)^{1/2}$

Subscript

0	= initial value

Introduction

WHEN a slender spinning vehicle enters the atmosphere at angle of attack, the resulting oscillations in pitch are rapidly damped and the envelope converges to some limiting value commonly referred to as the rolling trim angle. The latter is a function of the basic configurational asymmetry and the ratio of the spin rate to the critical frequency, which for slender vehicles is essentially the undamped frequency in pitch. Figure 1 is a convenient schematic showing the characteristic locus of rolling trim for a vehicle having a small nonzero trim angle. In the general vicinity of resonance $(0.95 < p/\omega_c < 1.05)$ the static value exhibits large amplification which can result in large normal loads and significant roll torques whenever the c.g. or c.p. is off the axis of symmetry. These torques may, in turn, produce large variations in the spin rate.

Spin behavior resulting from configurational asymmetry is of interest because it has been demonstrated by Pettus[1] that the slender entry vehicle is particularly sensitive to roll-pitch lock-in. When this occurs, the spin rate builds up in unison with the pitch frequency and a condition of persistent resonance then exists accompanied by large and possibly catastrophic excursions in angle of attack. It is promoted by one or more forms of inherent or acquired compound asymmetry in excess of some critical threshold level. Various criteria for its occurrence and avoidance have been developed in the literature.[1-6]

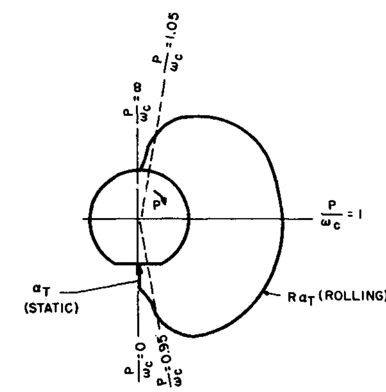

Fig. 1 Schematic of rolling trim orientation and amplification with p/ω_c.

This Note deals with the transient response of a vehicle in rolling trim at first intersection of the spin and pitch frequencies, and the susceptibility to steady resonance immediately thereafter for the two types of compound asymmetry shown in Fig. 2.

Transient Resonance Response

The characteristic excursion in angle of attack which occurs at the altitude of transient resonance is usually not of great concern. However, it does become important when considering resonance susceptibility, since the roll torques associated with this transient response will determine whether $\dot{p}$ exceeds $\dot{\omega}_c$, which is a necessary condition for lock-in.

It has been demonstrated analytically by Kanno[7] that the angle-of-attack response of a slender entry vehicle to transient resonant excitation can be conveniently expressed as a function of the nondimensional parameter λ, where

$$\lambda = p_0(1 - I_x/2I_y)/\beta V_E \sin\gamma_E \qquad (1)$$

The analysis presupposes a linear flight path, constant velocity, and constant roll rate p_0. A numerical solution for $|\alpha/\alpha_T|_{\max}$ which will be referred to as the amplification ratio

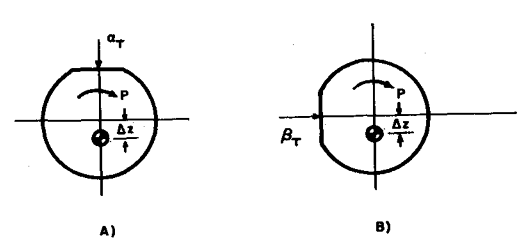

Fig. 2 Compound asymmetry models comprising radial c.g. offset and A) opposing coplanar trim, B) orthogonal leading trim.

Received May 7, 1970; revision received November 30, 1970. This study was performed in part under Air Force Contract AF04(694)-913 (RVTO). Originally written for presentation as Paper 68-23 of the Sixth Aerospace Sciences Meeting held in New York, January 22–24, 1968 but withdrawn for classification reasons.

* Group Leader, Vehicle Dynamics, Flight Mechanics Section. Member AIAA.

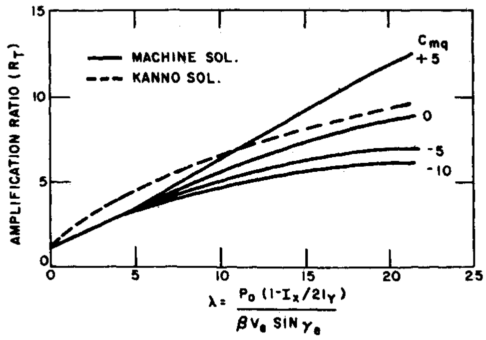

Fig. 3 Transient amplification ratio as a function of λ.

R_T, is given in the reference for the case where C_{m_q} is zero, and is shown in Fig. 3. One can curve fit this solution to obtain a simple expression for R_T thus:

$$R_T = 2(\lambda)^{1/2} \tag{2}$$

for $\lambda > 1$, which is perfectly acceptable since most entry vehicles of interest here have λ values in excess of 10.

If the vehicle has a negative C_{m_q} the amplification will be reduced and likewise the susceptibility to resonance. The converse is true if C_{m_q} is positive. In order to determine the effect of C_{m_q} on R_T, six-degrees-of-freedom digital computer studies have been performed[8] in which C_{m_q} has been varied parametrically. Several vehicles have been evaluated, and this has led to the generalized set of working curves for R_T shown in Figure 3. The Kanno solution is seen to be slightly conservative. The curves can be used to determine not only the angle-of-attack excursion induced by trim at transient resonance, but also the correction factor which must be applied to resonance criteria in order to reflect the C_{m_q} effect. This is discussed in the next section.

A Criterion for Steady Roll Resonance

An expression can now be developed for predicting the critical level of double asymmetry above which steady resonance will occur following transient resonance. The analysis will be applied to the case of a c.g. offset and an opposing coplanar trim and then to show that this solution is also closely applicable to the orthogonal leading trim case.

As stated previously, the necessary criterion is that $\dot{p}$ exceed $\dot{\omega}_c$. At initial resonance, with the rolling trim at 90° phase lag:

$$\dot{p} = C_{N\alpha}R_T(\alpha_T \cdot \Delta z)qS/I_x \tag{3}$$

One can express ω_c as a function of t and hence obtain its derivative by assuming a straight line flight path, a constant vehicle velocity equal to the entry value, and an exponential variation of density with altitude. This gives $\omega_c = (\omega_c)_0 \exp[\beta V_E \sin\gamma_E/2]$ whence:

$$\dot{\omega}_c = \omega_c \beta V_E \sin\gamma_E/2 \tag{4}$$

Applying the criterion and rearranging, it is required that

$$(\alpha_T \cdot \Delta z) \geq \omega_c I_x \beta V_E \sin\gamma_E/2C_{N\alpha}R_T qS \tag{5}$$

Substituting for R_T from Eq. (2):

$$(\alpha_T \cdot \Delta z) \geq \omega_c I_x(\beta V_E \sin\gamma_E)^{3/2}/4C_{N\alpha}qS[p_0(1 - I_x/2I_y)]^{1/2} \tag{6}$$

Thus, resonance will occur if the double asymmetry product exceeds the value of the expression on the right, which will be referred to as K_{cr}. The latter can be simplified by noting that at resonance $p = \omega_c$ and also that $C_{M\alpha} = -C_{N\alpha}\bar{X}/D$. Substitution and rearrangement of these terms leads to the result:

$$K_{cr} = (I_x/4I_y)(1 + 3I_x/4I_y)(\beta V_E \sin\gamma_E/p_0)^{3/2}\bar{X} \tag{7}$$

The term $(1 + 3I_x/4I_y)$ can be set equal to 1 so that

$$K_{cr} = (I_x/4I_y)(\beta V_E \sin\gamma_E/p_0)^{3/2}\bar{X} \tag{8}$$

Now consider the orthogonal case. Figure 1 shows that in the vicinity of resonance, with p/ω_c slightly less than unity, the amplified rolling trim is to all intents and purposes at zero phase lag with respect to its static location. To be conservative, the assumption is made that the full moment arm Δz is realized. This leads to the same expression for K_{cr}.

Equation (8) has been found to give good correlation with values for K_{cr} determined by analog and digital computer simulation. Typical results are shown in Fig. 4. As would be expected, a slightly greater level of orthogonal asymmetry can be tolerated than with coplanar before the onset of steady resonance.

The equation for K_{cr} is based on a curve fit for R_T which assumes zero C_{m_q}. Since steady resonance susceptibility varies directly with R_T, it is possible to obtain a simple correction for K_{cr} for nonzero values of C_{m_q} by means of the equation:

$$K_{cr}' = K_{cr} \cdot (R_T)_{C_{mq}=0}/(R_T)_{C_{mq}=0} \tag{9}$$

where R_T values are read directly from Fig. 3.

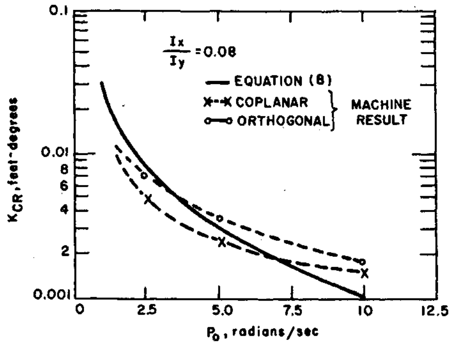

Fig. 4 Comparison of analytical prediction for K_{cr} with six-degrees-of-freedom computer results as a function of entry roll rate.

Effects of Products of Inertia

It can readily be deduced from the equations of motion that the inertial asymmetry terms I_{xy} and I_{xz} induce equivalent trim angles of the form:

$$\alpha_T = -(p/\omega_c)^2 I_{xz}/I_y - I_x$$
$$\beta_T = -(p/\omega_c)^2 I_{xy}/I_y - I_x \tag{10}$$

where $I_{xy}/I_y - I_x$ and $I_{xz}/I_y - I_x$ represent the absolute values of the components of principle axis tilt ϵ in the y and z directions. At resonance where $p = \omega_c$, the equivalent trim is equal to the principle axis tilt and this is the angle which is amplified.

Hence, a radial c.g. offset in combination with a principle axis component whose orientation is equivalent to one or other of the two compound asymmetry models considered previously might be expected to produce lock-in if the product exceeds K_{cr}. This does happen in the case of orthogonal combinations but not with the coplanar model. In the latter case, angle-of-attack histories obtained on the computer simply show an equivalence at transient resonance, but lock-in does not occur even when the equivalent trim is raised by an order of magnitude above its K_{cr} level. A satisfactory explanation is beyond the scope of this Note.

These considerations lead to the following mass balance criteria for the avoidance of high altitude steady resonance in a vehicle having a positive direction of spin:

$$P(I_{xy}\Delta z) \leq 0; \qquad P(I_{xz}\Delta y) \geq 0 \tag{11}$$

and if conditions of Eq. (11) cannot be met then,

$$\Delta r(\epsilon + \alpha_T) < K_{cr} \tag{12}$$

where α_T is some trim angle caused by external configurational asymmetry. At high altitude prior to the onset of significant ablation, this term is probably negligible. For negative spin directions, the inequality signs in Eq. (11) are reversed.

References

[1] Pettus, J. J., "Persistent Reentry Vehicle Roll Resonance," AIAA Paper 66-49, New York, 1966.

[2] Migotsky, E., "On a Criterion for Persistent Reentry Vehicle Roll Resonance," AIAA Paper 67-137, New York, 1967.

[3] Platus, D. H., "A Note on Reentry Vehicle Roll Resonance," *AIAA Journal*, Vol. 5, No. 7, July 1967, pp. 1348–1350.

[4] Price, D. A., Jr., "Sources, Mechanisms, and Control of Roll Resonance Phenomena for Sounding Rockets," *Journal of Spacecraft and Rockets*, Vol. 4, No. 11, Nov. 1967, pp. 1516–1525.

[5] Price, D. A., Jr. and Ericsson, L. E., "A New Treatment of Roll-Pitch Coupling for Ballistic Re-Entry Vehicles," *AIAA Journal*, Vol. 8, No. 9, Sept. 1970, pp. 1608–1615.

[6] Barbara, F. J., "An Analytical Technique for Studying the Anomolous Roll Behavior of Reentry Vehicles," *Journal of Spacecraft and Rockets*, Vol. 6, No. 11, Nov. 1969, pp. 1279–1284.

[7] Kanno, J. S., "Spin-Induced Forced Resonant Behavior of a Ballistic Missile Reentering the Atmosphere," LMSD-288139, Vol. III, Jan. 1960, Lockheed Aircraft Corp., Missiles & Space Div., Sunnyvale, Calif.

[8] Bootle, W. J. and Kuczkowski, T., "Effects of C_{mq} on Transient Resonance Trim Amplification," TR-S240-67-WB-TK-70, June 1967, Avco Systems Div., Wilmington, Mass.

Application of Transformation Methods to Wedge-Shock/Boundary-Layer Interactions

DONALD R. KOTANSKY*
General Dynamics, Fort Worth, Texas

THE need for a method for the prediction of changes in the properties of the compressible turbulent boundary layer in the region of interaction with wedge or corner-induced oblique shock waves led to the development of the method presented here. The successful application of compressible boundary-layer transformation techniques to related problems by many authors, notably the application of boundary-layer transformation techniques to incident-reflecting-shock-wave/boundary-layer interaction problems by Seebaugh, Paynter, and Childs,[1] suggested the application of this technique to the wedge-shock/boundary-layer interaction problem.

General Properties of the Power-Law Transformation

The basic relationships are derived following the analysis of Ref. 1. The physical coordinates are transformed to a new set of coordinates for a corresponding incompressible flow

$$X(x) = x \tag{1}$$

$$Y(x,y) = \int_0^y \rho/\rho_e \, dy \tag{2}$$

It is assumed that the stream function, ψ, is invariant under the transformation and that the physical compressible boundary layer velocity profile may be expressed as a typical turbulent power-law profile, i.e.,

$$u/u_e = (y/\delta)^{1/n} \tag{3}$$

A similar relationship is assumed for the transformed flow

$$U/U_e = (Y/\Delta)^{1/N} \tag{4}$$

The assumption of constant energy in the boundary layer requires that the local stagnation enthalpy in the boundary layer be equal to the freestream stagnation enthalpy. For a perfect gas, this results in a simple relationship between local velocity and static temperature. The maximum attainable velocity, u_{max}, is that velocity corresponding to zero static temperature. If ϕ is defined as the ratio of u_e to u_{max}, this relationship becomes

$$T = [1/\phi^2 - (u/u_e)^2]u_e^2/2C_p \tag{5}$$

Through the use of the equation of state for a perfect gas and the assumption that the static pressure is constant through the boundary-layer thickness,

$$\delta = \int_0^\Delta T/T_e dY \tag{6}$$

Substitution of Eq. (5) into Eq. (6) and evaluation of the integral yields

$$\delta = D\Delta \tag{7}$$

where

$$D = (1 - \phi^2)^{-1}[1 - N\phi^2/(2 + N)] \tag{8}$$

From the relation between the physical and the transformed velocity and the definition of the stream function,

$$U = (\partial\Psi/\partial Y) = (\rho_e/\rho_0)u = (\rho_e/\rho)(\partial\psi/\partial y) \tag{9}$$

The invariance of the stream function requires that

$$\Psi(Y = \Delta) = \psi(y = \delta) \tag{10}$$

From Eqs. (9) and (10),

$$\int_0^\Delta \rho_0 U dY = \int_0^\delta \rho u dy \tag{11}$$

Evaluation of the integral on the left-hand side of Eq. (11) and further simplification yields

$$\frac{N/(1 + N)}{[1 - N\phi^2/(2 + N)]} = \int_0^1 \frac{(y/\delta)^{1/n} \, d\left(\frac{y}{\delta}\right)}{[1 - \phi^2(y/\delta)^{2/n}]} d\left(\frac{y}{\delta}\right) \tag{12}$$

Evaluation of the integral in Eq. (12) for various Mach numbers and compressible power-law exponents, $1/n$, yields the results shown in Fig. 1.

Wedge-Shock/Boundary-Layer Interaction

The control volume selected along with other necessary geometric quantities for the two-dimensional wedge-shock/boundary-layer interaction model is shown in Figure 2. The streamwise boundaries of the control volume are the compression surface downstream of the corner and a streamline passing through the corner shock in the inviscid region of the flow. The flow in the boundary layer passing through the control surfaces is assumed to be parallel to the compression surfaces. This precludes the existence of large flow disturbances in the boundary layer upstream of the compression corner; this is substantiated by experimental data in Ref. 2 where it was found that, in the absence of separation, the upstream influence was considerably less in extent than a length equivalent to one boundary-layer thickness. This requirement and the assumption of negligible wall shear stresses on the compression surface downstream of the compression corner within the interaction control volume are the most significant simplifications in the formulation of the model.

Received May 4, 1970; revision received December 9, 1970. The author gratefully acknowledges the assistance of T. R. Slaten and T. W. Harmon in performing the necessary calculations.

* Project Propulsion Engineer. Now Group Engineer-Propulsion, McDonnell Aircraft Company, St. Louis, Mo. Member AIAA.